8
The tropical Hadley circulation

The gap between simulation and understanding
On the one hand we try to simulate by capturing as much of the dynamics as we can in comprehensive numerical models. On the other hand, we try to understand by simplifying and capturing the essence of a phenomenon in idealized models… I.M. Held, 2005: The gap between simulation and understanding in climate modelling. **Bull.Am.Meteorol.Soc., 86**, 1609-1614.

8.1 Introduction

This short chapter is devoted to the dynamics of the well-known tropical tropospheric Hadley circulation (**figure 8.1**). Ideally, the Hadley circulation is a zonally symmetric (independent of longitude) tropospheric circulation, which is driven by the meridional gradient in diabatic heating, which is dominated by latent heat release in clouds over the ITCZ. This creates a diabatic heating surplus in the tropics, which disturbs the state of thermal wind balance. The atmosphere strives to maintain thermal wind balance by creating a vertical meridional circulation, with poleward flow at upper levels and equatorward flow at lower levels, which changes the zonal mean thermal wind [1] and the zonal mean meridional temperature gradient such that thermal wind balance is preserved. This process is captured very succinctly by the the "**Held-Hou" model** (Held and Hou, 1980).

8.2 Hou-Held theory of the The Hadley-circulation and the subtropical jet

How does a vertical meridional circulation alter the vertical shear of the zonal wind? This can be easily understood on the basis of the law of **conservation of angular momentum per unit mass**, M_a, of a zonal ring of air. If the ring of air is at a latitude, ϕ, then

$$\boxed{M_a = (\Omega a\cos\phi + [u])a\cos\phi} \quad . \tag{8.1}$$

[1] The zonal mean thermal wind is defined the vertical shear of the geostrophic zonal wind

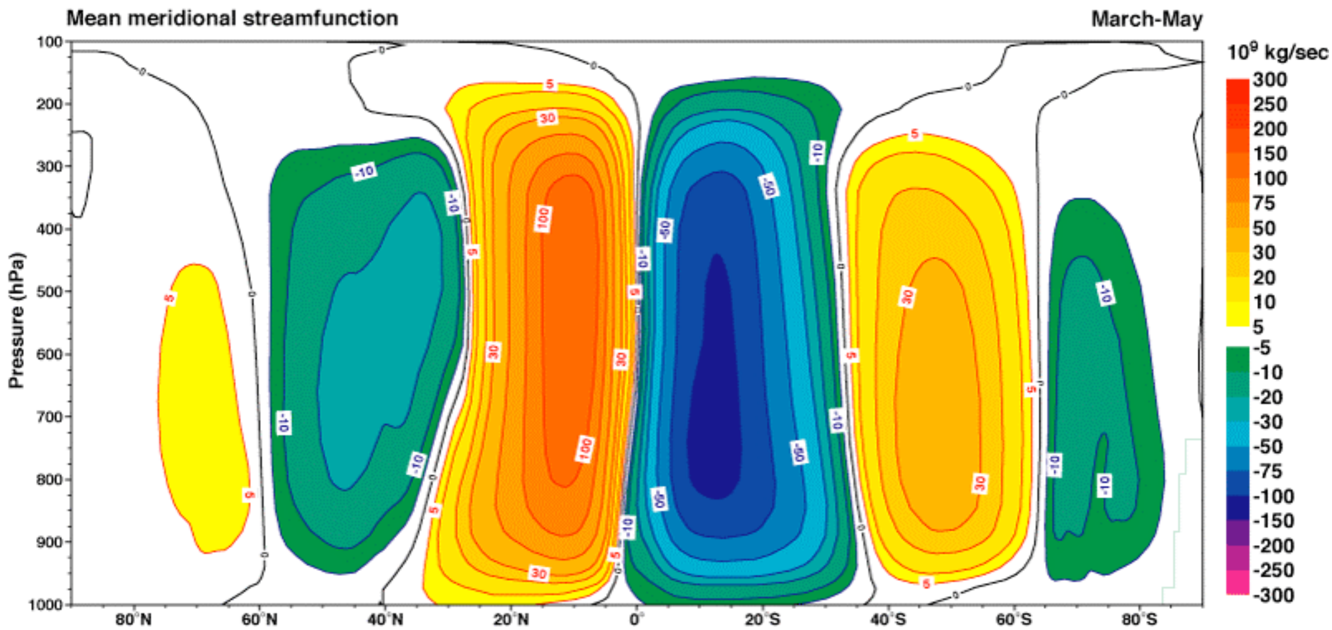


FIGURE 8.1. Longitudinally averaged meridional circulation averaged over the months of March, April and May of the years 1979-2001, according the the ERA-40 (ECMWF) reanalysis. The two cells on either side of the equator are termed "Hadley" cells. The mid-latitude- and polar cells are not discussed in this section. Source: http://www.ecmwf.int/research/era/ERA-40_Atlas/index.html.

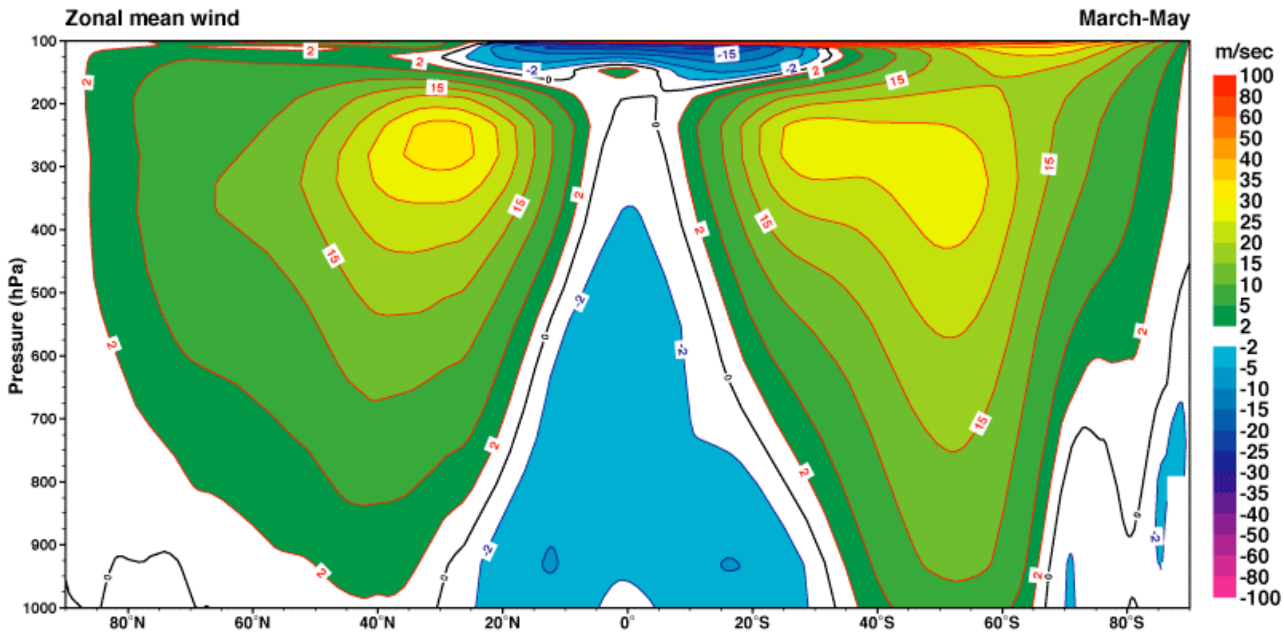


FIGURE 8.2. Longitudinally averaged zonal wind averaged over the months of March, April and May of the years 1979-2001, according the ERA-40 (ECMWF) reanalysis. In the northern hemisphere a **subtropical jet** is observed in the upper troposphere at about 30°N and 250 hPa. Source: http://www.ecmwf.int/research/era/ERA-40_Atlas/index.html.

In eq. 8.1 $[u]$ is the longitudinally averaged zonal velocity (a function of the meridional coordinate, y), Ω is the angular velocity of the Earth and a is the radius of the Earth. If $[u]$=0 at the equator then conservation of M_a requires that

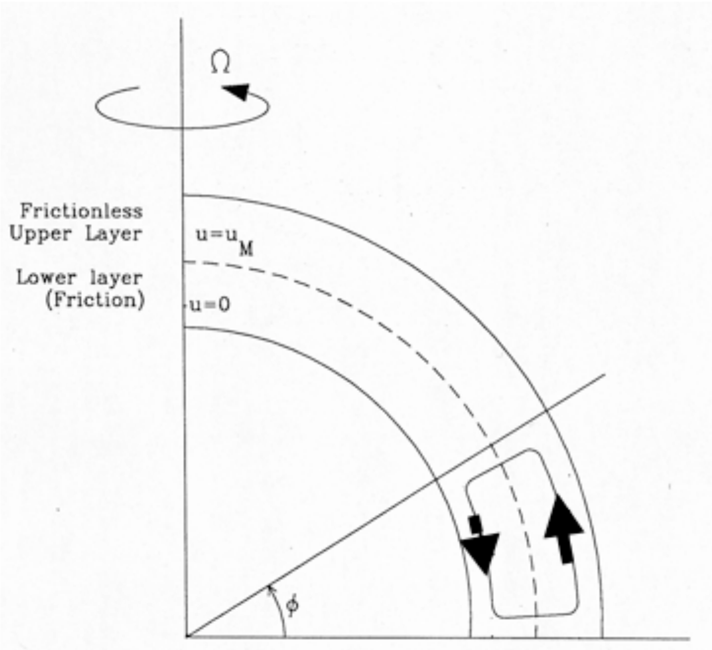


FIGURE 8.3. Schematic illustration of the Held-Hou model of the Hadley circulation at equinox. In this **figure** *u* is the zonal velocity, averaged along a latitude circle. Based on **figure 4.4 in James (1994)** (**Introduction to Circulating Atmospheres**. Cambridge University Press, 422 pp.).

$$[u] = \frac{\Omega a \sin^2\phi}{\cos\phi} \approx \frac{\Omega y^2}{a\cos\phi} \qquad (8.2)$$

at any other latitude, ϕ, assuming that air parcels travel from the equator polewards. The square brackets in equation (8.2) indicate averaging along a full latitude circle. Eq. 8.2 implies that $[u]$ increases rougly in a quadratic fashion away from the equator. According to eq. 8.2°, $[u]$=127 m s^{-1} at a latitude of ϕ=30! At more poleward latitudes, the air parcels coming from the equator would attain even larger zonal velocities. These velocities are actually not observed (**figure 8.2**), which indicates that angular momentum is not conserved, certainly not at latitudes polewards of about 25 or 30° (why not?). Due to the poleward flow at upper levels and the simultaneous equatorward flow at lower levels, and because angular momentum is conserved, the vertical shear of the zonal wind increases so that it "comes into thermal wind balance" with the meridional temperature gradient according to the thermal wind equation:

$$\boxed{2\Omega\sin\phi\frac{\partial[u]}{\partial z} = -\frac{g}{\theta_0}\frac{\partial[\theta]}{\partial y}} \qquad (8.3)$$

(**section 1.19**). In a celebrated paper, published in 1980, Held and Hou (see the reference list at the end of this chapter) formulated a **model of the Hadley circulation** based on angular momentum conservation and simultaneous maintainance of themal wind balance.

Held and Hou assume that the structure of the atmosphere that can be represented by two layers (**figure 8.3**): an upper outflow layer, where the *zonal* velocity obeys angular momentum conservation and a lower inflow layer where the *zonal* velocity component is so small, that it can be assumed to be equal to zero. The latter assumption is rather drastic and of course not realistic. Nevertheless, in reality, the vertical wind shear will be determined mainly by the wind in the upper layer. If the poleward flow in the upper layer takes place at height H, we may approximate the vertical shear of the zonal wind by

$$\frac{\partial[u]}{\partial z} \approx \frac{U_M}{H} , \tag{8.4}$$

where U_M represents the zonal velocity in the upper layer (the subscript M stands for angular momentum conservation). Thus (eq. 8.2),

$$\boxed{U_M \approx \frac{\Omega y^2}{a\cos\phi}} . \tag{8.5}$$

Substituting (8.5) into (8.4) and the result into (8.3) we get

$$\boxed{\frac{\partial[\theta]}{\partial y} = -\frac{2\Omega^2\theta_0}{a^2 gH} y^3} . \tag{8.6}$$

We have derived an expression for the meridional gradient of the zonal mean potential temperature, which is **required to conserve angular momentum and at the same time maintain thermal wind balance**.

Integration of eq. 8.6 with respect to y gives the associated zonal mean potential temperature:

$$\theta_M = \theta_{M0} - \frac{\Omega^2\theta_0}{2a^2 gH} y^4 . \tag{8.7}$$

Here, we have dropped the square brackets; the subscript M is used again to indicate that this is the **potential temperature that is required by angular momentum conservation**. The parameter θ_{M0} a constant of integration, which must interpreted as the equatorial temperature.

The potential temperature distribution, which is imposed by angular momentum conservation, need not be consistent with the potential temperature distribution, which is imposed by diabatic heating. Held and Hou (1980) neglected the effect of latent heating and assumed that diabatic heating is due to radiative imbalance. If the actual potential temperature does not depart too much from the radiative equilibrium potential temperature (section 2.4), the heating due to this radiative imbalance can be parametrised according to the following equation.

$$\boxed{\frac{d\theta}{dt} = -\frac{\theta - \theta_E}{\tau_E} = -\frac{\theta_M - \theta_E}{\tau_E}} . \tag{8.8}$$

Here, θ_E is the **radiative equilibrium potential temperature**, i.e. the potential temperature that is established by radiative processes. This might be the **radiative equilibrium temperature** or the **radiatively determined temperature**. The radiative equilibrium temperature is the temperature that would result due to radiative fluxes in the absence of motion and radiative flux divergence. The radiatively determined temperature is the temperature that would result due to radiative fluxes in the absence of motion, but allowing for radiative flux divergence due to the thermal inertia of the atmosphere and the earth's surface (section 2.4). Because of the relatively small amplitude of the annual cycle in insolation in the tropics, these two potential temperature distributions do not differ much in the tropics.

The parametrisation of diabatic heating according to (8.8) is frequently referred to as "**Newtonian heating or cooling**". In eq. 8.8, τ_E is referred to as the **radiative relaxation time**, or, as in section 2.4, the radiative equilibrium timescale. In the annual mean (or at equinox), both θ_E and θ_M possess a maximum at the equator. Usually, $(\theta_M-\theta_E)<0$ at the equator, implying heating, while $(\theta_M-\theta_E)>0$ at more poleward latitudes, implying cooling.

Held and Hou (1980) assumed, rather arbitrarily, that

$$\theta_E = \theta_0 - \frac{2}{3}\Delta\theta P_2(\sin\phi) \tag{8.9}$$

where P_2 is a second legendre polynomial:

$$P_2(x) \equiv \frac{1}{2}\left(3x^2 - 1\right), \tag{8.10}$$

and $\Delta\theta$ is the equator-pole temperature difference in the radiative equilibrium. Since

$$\sin\phi \approx \frac{y}{a} \tag{8.11}$$

for small values of ϕ,

$$\theta_E \approx \theta_{E0} - \frac{\Delta\theta}{a^2}y^2 \ . \tag{8.12}$$

So, we see that θ_E has a quadratic dependence on y, while θ_M has a quartic dependence on y, A graph of the actual potential temperature, θ_M, and of the radiative equilibrium potential temperature, θ_E, is shown in **figure 8.4**. The poleward motion ceases at a latitude, $\phi_p=Y/a$. This latitude is determined by choosing θ_{M0} so that there is no net heating within the Hadley circulation. Assuming that τ_E does not vary with latitude this implies that

$$\int_0^Y (\theta_M - \theta_E)dy = 0 \ . \tag{8.13}$$

In other words,

$$\theta_{M0} - \frac{\Omega^2\theta_0}{10a^2gH}Y^4 = \theta_{E0} - \frac{\Delta\theta}{3a^2}Y^2 \ . \tag{8.14}$$

Furthermore, since $\theta_M=\theta_E$ at $y=Y$,

$$\theta_{M0} - \frac{\Omega^2\theta_0}{2a^2gH}Y^4 = \theta_{E0} - \frac{\Delta\theta}{a^2}Y^2 \ . \tag{8.15}$$

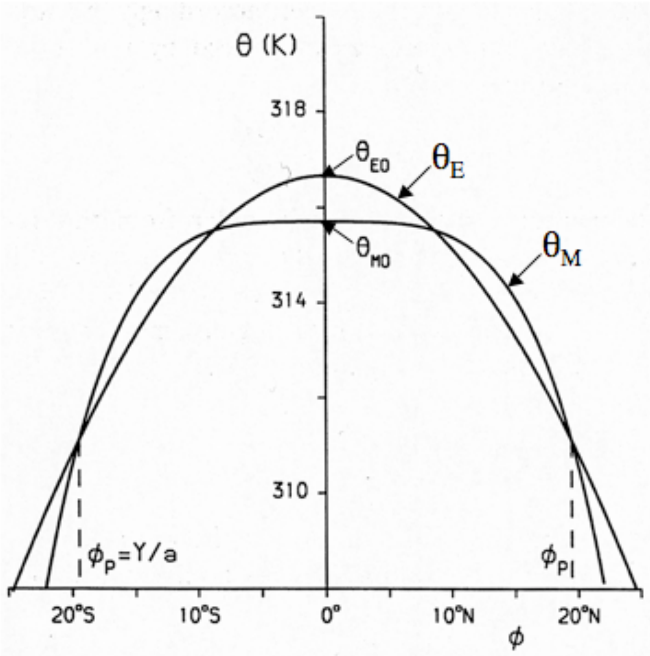


FIGURE 8.4. The radiative equilibrium potential temperature and the actual potential temperature (required by both angular momentum conservation and thermal wind balance) as a function of latitude. Based on figure 4.5 in James (1994) (**Introduction to Circulating Atmospheres**. Cambridge University Press, 422 pp.).

From (8.14) and (8.15) we get the following expression for the latitudinal extent of the Hadley circulation

$$\boxed{Y = \sqrt{\left(\frac{5gH\Delta\theta}{3\Omega^2\theta_0}\right)}} \, . \qquad (8.16)$$

With θ_0=255 K, H=10 km and $\Delta\theta$=40 K we find Y=2200 km, which is roughly in agreement with observations. The latitudinal extent of the Hadley circulation is inversely proportional to the angular velocity of the Earth. This explains the fact that the poleward limit of the Hadley-like circulation on the planet Venus, which rotates much more slowly than Earth, is about ±60° latitude.

Furthermore, from (8.14) and (8.15) we deduce that

$$\theta_{E0} - \theta_{M0} = \frac{5gH\Delta\theta^2}{10a^2\Omega^2\theta_0} \, . \qquad (8.17)$$

At the equator the difference between the actual temperature and the radiative equilibrium temperature is sustained by the adiabatic temperature decrease due to upward motion in a stably stratified atmosphere. Assuming a steady state:

$$w\frac{\partial\theta}{\partial z} = \frac{\theta_{E0} - \theta_{M0}}{\tau_E} \, , \qquad (8.18)$$

or

$$w \approx \frac{g}{\theta_0 N^2} \frac{\theta_{E0} - \theta_{M0}}{\tau_E} \ . \tag{8.19}$$

Taking τ_E as 15 days (section 2.4) and N as 10^{-2} s^{-1}, we find that w is about 0.24 mm/s. From continuity we can estimate the magnitude of the horizontal velocity as

$$v \approx \frac{Y}{H} w \approx 0.5 \text{ cm s}^{-1}. \tag{8.20}$$

The observed meridional winds are in the order of 1 m s^{-1}. Therefore, the Held-Hou model has provided a reasonable estimate of the geometry of the Hadley cell, but a very poor estimate of its strength. This is probably due to the neglect of latent heat release due to condensation of water vapour in clouds in the ITCZ, which in reality is certainly the principal heat source, which drives the Hadley circulation.

Despite its relative lack of realism the Held-Hou model provides valuable "first order" insight into the essential physics of the Hadley circulation, namely that it exists because the atmosphere wants to adjust to thermal wind balance, but can never attain exact thermal balance because it is forced to simultaneously satisfy two slightly incompatible constraints, i.e. angular momentum conservation drives the atmosphere towards a temperature that usually differs from the diabatically determined temperature.

PROBLEM 8.1. Latitude dependence of the radiative equilibrium temperature at equinox

Using the theory in **Boxes 2.1** and **2.4**, determine the approximate dependence on y (latitude) at different isobaric levels in the atmosphere (e.g. at 500 hPa and at 300 hPa and) of the *radiative equilibrium temperature at equinox* in an atmosphere with one well mixed greenhouse gas. In which way might this change if the effect of water vapour as a greenhouse gas is taken into account, and what consequences might this have for the Hadley circulation, if we believe the Held-Hou model?

PROBLEM 8.2 Extension of the Held-Hou model to the case of heating centred off the equator

Lindzen and Hou (1988) (see also James (1994) in the list of reference at the end of this chapter) extended the Held-Hou model to the more realistic case involving heating centred off the equator (such as during solstice). What is the consequence for the curve of θ_M (eq. 8.5) if the maximum of θ_E is located at latitude of 10N° (see **figure 8.4**)?

ABSTRACT OF CHAPTER 8

The atmosphere maintains thermal wind balance by rearranging mass and momentum through a **vertical and meridional circulation**, sometimes called, **"secondary circulation"**. An example of a secondary circulation is the **Hadley circulation** in the tropics, which represents the dynamical response to diabatic heating, needed to maintain thermal wind balance. The diabatic heating, which depends on the existence of the circulation, drives the atmosphere away from wind balance. The **Hou-Held model** succinctly illustrates this process.

Further reading

Textbook

James, I.N., 1994: **Introduction to Circulating Atmospheres**. Cambridge University Press. 422 pp. (Held-Hou theory of the Hadley circulation: section 4.2).

Article

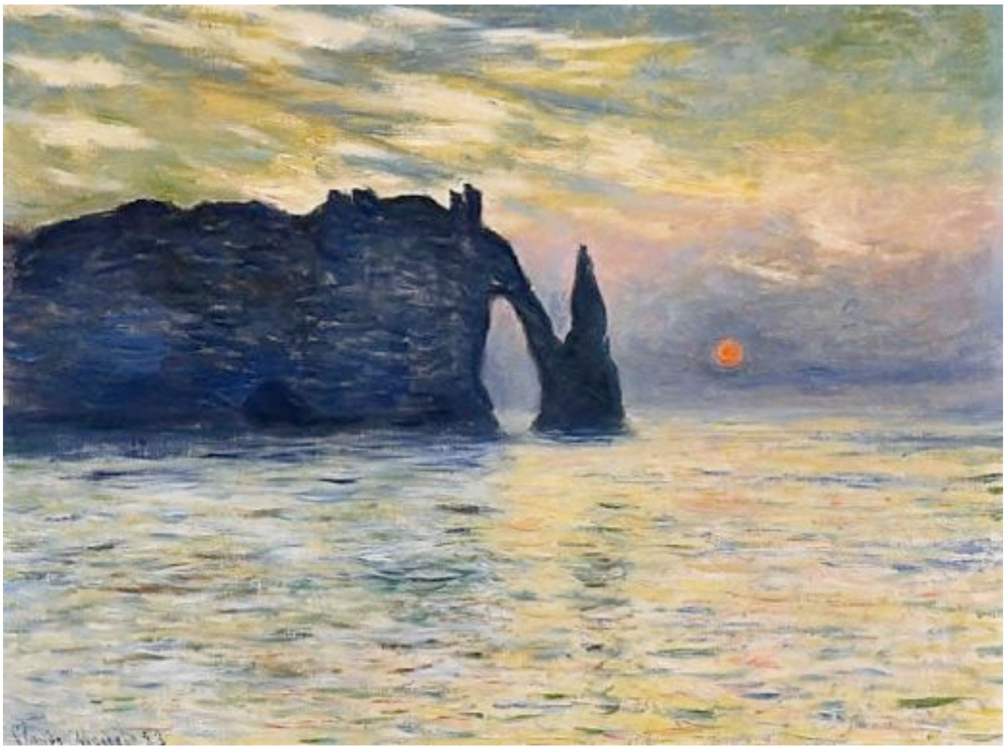
Held, I.M., and A.Y. Hou, 1980: Nonlinear axially symmetric circulations in a nearly inviscid atmosphere. **J.Atmos.Sci.**, 37, 515-533.

Lindzen, R.S. and A.Y. Hou, 1988: Hadley circulations for zonally averaged heating centred off the equator. . **J.Atmos.Sci.**, 45, 2416-2427.

List of problems (chapter 8)

This is the May 2018 edition of chapter 8 of the lecture notes on Atmospheric Dynamics (first written in 2003), by Aarnout van Delden (IMAU, Utrecht University, Netherlands, a.j.vandelden@uu.nl).

http://www.staff.science.uu.nl/~delde102/AtmosphericDynamics.htm



"Le soleil couchant" by Claude Monet, North Carolina Museum of Art. High layered clouds hint at an approaching warm front at sunset in Etretat (Normandy, France).